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Assessing The Characteristics of Elementary School Students' Mental Models in Understanding The Concept of Integers

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Abstrak

Penelitian ini bertujuan untuk mengungkap karakteristik model mental siswa sekolah dasar dalam memahami konsep dari bilangan bulat. Subyek dalam penelitian ini dipilih berdasarkan kriteria outlier atau tanggapan unik terhadap serangkaian tes yang diberikan kepada 30 siswa kelas tiga sekolah dasar (SD). Berdasarkan hal tersebut, ada dua siswa yang dijadikan subjek penelitian. Data penelitian dikumpulkan melalui observasi dan wawancara. Pengamatan yang dilakukan peneliti adalah pengamatan langsung, yaitu peneliti melihat dan mengamati aktivitas siswa yang mengerjakan soal tes. Selain itu, peneliti melakukan wawancara dengan subjek penelitian. Hasil pekerjaan siswa direview dan setelahnya dilakukan wawancara untuk memperjelas jawaban siswa. Hasil penelitian menunjukkan bahwa karakteristik siswa sekolah dasar dalam memahami bilangan bulat tidak selalu berpindah dari tahap sintetik ke tahap formal, tetapi terdapat siswa kelas rendah yang mampu menyelesaikan masalah kontekstual terkait dengan bilangan bulat. Proses pembentukan model mental formal siswa dapat dilihat dengan memecahkan masalah kenaikan suhu dengan menghitung jari dan membandingkannya dengan posisi simetri nol antara bilangan bulat negatif dan positif.

Kata Kunci: penilaian, karakteristik model mental, konsep bilangan bulat.

Abstract

This study aims to reveal the characteristics of the mental model of elementary school students in understanding the concept of integers. The subjects in this study were selected based on outlier criteria or unique responses to a series of tests given to 30 students of the third grade of elementary school. As a result, two students were chosen as research subjects. The research data were collected through observation and interviews. Observation made by the researchers was direct observations, in which the researchers see and observe the activities of students in doing the test. In addition, the researchers also conducted interviews with research subjects. The results of the students' works were reviewed, and then interviews were conducted to clarify the students' answers. The results showed that the characteristics of elementary school students in understanding integers did not always move from the synthetic stage to the formal stage, but there were lower grade students who were able to solve contextual problems related to integers. The process of forming students' formal mental models can be seen when they solve the temperature rise problem by counting fingers and comparing them with the zero symmetry position between negative and positive integers.

Keywords: *assessment, characteristic of mental model, the concept of integers.*

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INTRODUCTION

Integers are topics that most students in the lower grades or primary school grades 1, 2, and 3 still find challenging (Muslimin, Putri, & Somakim, 2012). According to Wahid, Agung, & Mirza (2015); Aksoy & Yajlik (2017); (Roselizawati, Sarwadi, & Masitah, 2014); (Fariha, 2019), most errors children make when solving school arithmetic problems arise when doing fundamental mathematical operations on integers. On the other hand, integer counting is the most critical and essential aspect of resolving classroom math issues (Wildaniati, 2015). Thus, one could argue that the idea of integers is one of the elements that pupils continue to find challenging, even though it is a critical component of the mathematics studied in school.

According to Lestari (2017), children may have difficulty counting operations because they misunderstand basic school math concepts. Students who can develop concepts and link them together to generate knowledge can tackle this challenge (Saguni, 2019). In order for students to construct an idea, they must modify their conceptual framework. So learning that builds concepts using student information can assist students in resolving those issues.

Alternatively, to assist students in formally building mathematical ideas, significant information regarding the underlying structure of student understanding from which knowledge is created may be extracted (Stafylidou & Vosniadou, 2004). Students frequently build conceptions throughout this process as they seek to combine new knowledge into conceptual frameworks that give insight into how they think about certain topics, referred to as mental models (Vosniadou & Brewer, 2004). The mental model is the point at which new knowledge is integrated into the knowledge base. Students modify their initial conceptions in response to instructor teaching, but they lack the supporting information necessary to comprehend new concepts. Additionally, (Özdemir & Clark, 2007) highlighted that teacher teaching leads lower-grade pupils to construct synthetic mental models that remain consistent with the newly acquired information. On the other hand, synthetic mental models indicate that pupils begin synthesizing prior information with new knowledge. As a result, assessing the features of low-grade pupils' mental models becomes critical for formally building mathematical concepts.

Assessment, according to (Mardapi, 2008), is a factor in determining educational quality. When teachers know their students' basic notions, it is easier for them to help students build on those foundational concepts as they progress to more complex ones (Vosniadou & Brewer, 2004). Students need their mental models assessed so that teachers may develop effective learning techniques to develop more complex concepts and receive high-quality instruction.

Some researchers categorize a person's mental models based on a variety of features to understand them better. Mental models have been studied by Barsalou (1992); Jaber & Boujaoude (2012); Park & Light (2009); Stafylidou & Vosniadou (2004); Bofferding (2014). Barsalou (1992) distinguishes between structural and conceptual mental models when describing the features of mental models. According to Jaber & Boujaoude (2012), there are three levels of mental models: high, medium, and low. The features of mental models are classified into five categories by Park & Light (2009). These include the initial formless or unclear mental models, the intermediate mental models 1, intermediate mental models 2, intermediate mental models 3, and the target mental models. Mental models are divided into three categories by Stafylidou & Vosniadou (2004): initial, synthetic, and formal mental models. With the help of Vosniadou (1992) as a starting point, Bofferding (2014) carried out research on the mental model by adding transitions 1 between the initial mental model and transition 2 between the synthetic and formal mental models. Because they cover structural mental models, conceptual mental models, early, synthetic, and formal mental models, the features of Bofferding's (2014) mental model accommodate many mental model investigations. In order to maximize students' mathematical learning capacity, according to Bofferding (2014), further study on mental models is required.

As a solution to negative integer reduction issues, Bofferding & Enzinger (2017) investigate how students link their logical reasoning to circular logical thinking. They also propose an additional study on whether or not

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all college students build formal mental models (Bofferding & Enzinger , 2017)). As a result, the objective of this research is to determine whether or not elementary school students in grades 1, 2, and 3 create formal mental models of integers before they reach the stage of the synthetic mental model, and how is the process of forming the formal mental model developed by the students.

METHOD

This study employed a qualitative case study. Two students were chosen for the study from a group of thirty third-grade elementary school students. The research participants were chosen from a pool of 30 elementary school students' responses based on unique criteria for their answers or different from other students' overall responses. Additionally, students who have not studied the idea of integers were excluded, as the study's objective was to determine whether any students acquired a formal mental model of integers prior to developing a synthetic mental model.

Researchers collected the data through observation and interviews. Direct observation by the researchers occurred when the researchers saw and observed students when completing exam tasks. The interview was then conducted on the research subject. The data for this study came from students' written responses after they developed the concept of integers and from researcher-conducted interviews. Each interview is videotaped, and students' written work is gathered. Three school mathematics teachers and two lecturers in mathematics education assessment validated the test question. Five items are judged valid based on their V'Aiken index of greater than 0.8 (Aiken, 1985); (Zainudin, Subali, & Jailani, 2019). The validity of the research was established by the analysis of computed data obtained from repeated watching and sharing of video data, as well as through the comparison and differentiation of data acquired from multiple data sources (Golafshani, 2003); (Mathison, 1988).

Analytic models were used to evaluate video data. Each interview video was examined twice in this methodology. All videos were carefully analyzed during the first cycle, paying particular attention to the link between the student's reaction and the relevant literature. The second cycle was done to identify critical events (Yilmas, 2017). These significant occurrences demonstrate the mental capacity of children's models. Once a critical event has been identified, the process of categorizing the data begins. This classification phase is critical in data analysis because it enables the meaningful interpretation of data. This classification is derived through a review of the literature, identifying important events that symbolize related concepts, and discussing these critical events and categorizations with other researchers. The coding methodology and description for each category are shown in Table 1.

Tabel 1
Deskripsi dari level model mental

Main Category	Description	Subcategory
Initial Mental Model	The child's early mental model form can be seen from their assumptions when they are not confronted with information from adults about a particular concept.	The subject does not yet understand negative integers; he considers negative integers to have the same value as positive integers.
Transition I	The mental model form of transition 1 can be seen from the imperfections of the child's efforts in aligning his ideas with information from adults about a particular concept.	Subjects are inconsistent in comparing two positive integers and negative integers
Synthetic Mental Model	The synthetic mental model form of a child can be seen from the child's attempts to reconcile his ideas with	Subjects can compare two negative numbers.

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Main Category	Description	Subcategory
	information from adults about a particular concept.	
Transition II	The mental model form of transition 2 can be seen when the child has not rearranged his or her theoretical framework and accommodated new information to reflect an adult's understanding of a particular concept.	Subjects are inconsistent in sorting positive integers and negative integers
Formal Mental Model	A child's formal mental model form can be seen when they successfully rearrange their theoretical framework and accommodate new information to reflect an adult's understanding of a particular concept.	The subject is able to understand the symmetry of negative integers

Source: (Utami, Sa'dijah, Subanji, & Irawati, 2018)

RESULTS AND DISCUSSION

Based on the results of a review of material in the school of research subjects, the third grade of elementary school has not been introduced to the concept of negative numbers. The material presented to the third-grade students is a positive integer and determines the pattern of numbers and the numbers between two given numbers. Subjects were asked to define negative numbers as well as differences in positive and negative numbers. He mentioned that negative numbers are low numbers with a "-" sign character (he calls them *"min"*) and positive numbers are high numbers. The following answer snippet indicates this.

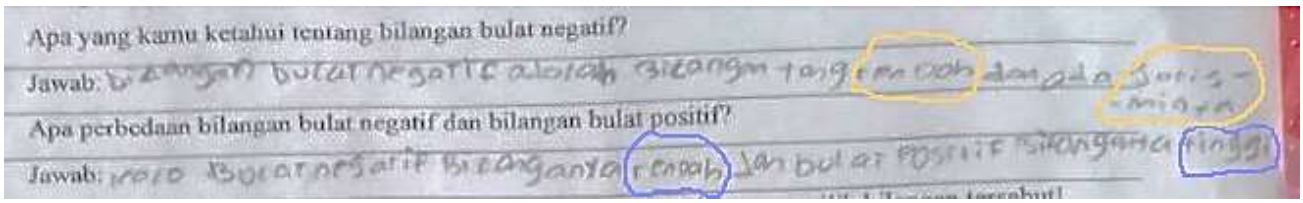

Figure 1. The response of the research subject on item 1

Translation:

What do you know about negative integers?

Answer: Negative integers are low numbers and have 'min' symbol

What is the difference between positive and negative integers?

Answer: The negative integers are low numbers, while positive integers are high numbers

When the subject was asked about the meaning of low and high numbers, he replied that positive numbers are greater than negative numbers. The subject holds the "-" sign (he calls it min) as a characteristic of a negative number. Then, the researchers examined what the sign "min" meant by asking, for example, 5 - 1 = ... Is 1 also a negative number? The subject responded differently, arguing that the sign "-" was a subtraction operation. This is evident from the following interview excerpt.

T: "What do you mean by low numbers and high numbers?"

S: "That's mom, the positive numbers are greater than the negatives."

T: "Low, high, big, small, what do you mean?"

S: "Well, the positive must be greater than negative, if there is a negative sign"

T: "What about 5 - 1 = ... Is 1 also a negative number? Is there a negative sign?"

S: "No, it's different, it's minus, Mom."

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Based on the test results and the interview excerpt above, the subject understands that a positive integer is a high number, while a negative integer is a low number and is marked with a "-" sign. The subject also states that "min" as a characteristic of negative numbers differs in meaning from the subtraction operation (-) on an integer operating system.

The subject was able to compare two negative numbers and positive numbers but could not yet compare two negative numbers. This is shown in the following answer snippet.

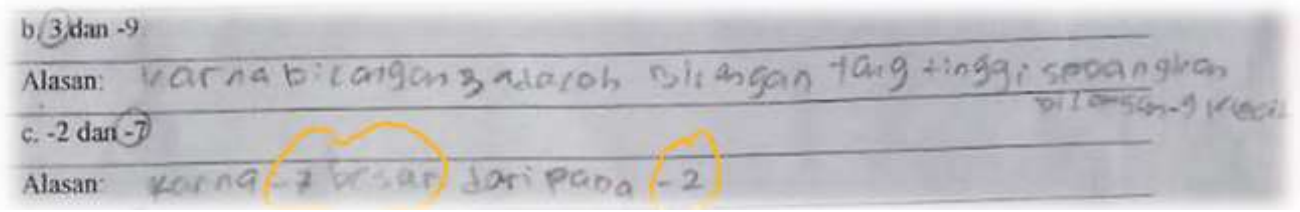

Figure 2. The response of the research subject on item 2

When the subject was asked why -7 was greater than -2, he replied because seven is greater than two. S1 ignores the "-" sign in front of the numbers 7 and 2. He considers the value of a negative number to be equal to a positive number. Here is the interview excerpt.

T: "Why is -7 greater than -2?"

S: "Yes, obviously, Ma'am, 7 is greater than 2."

T: "Then what is the difference between 7 and -7?"

S: "emm… it's preceded by *min* and another is not."

Based on the results of the tests and interviews above, the subject already understood the comparison between positive integers and negative integers. But the subject has not been able to compare two negative integers. This is because the understanding of the subject who considers a positive integer is a high number and a negative integer is a low number. So when comparing two negative numbers, the subject rates the same when comparing two positive integers (or regardless of the negative sign in front of the number).

The subject also could not sort the negative integers correctly. This is because the subject views the value of a negative integer is equal to a positive integer. Here is the snippet of the answer.

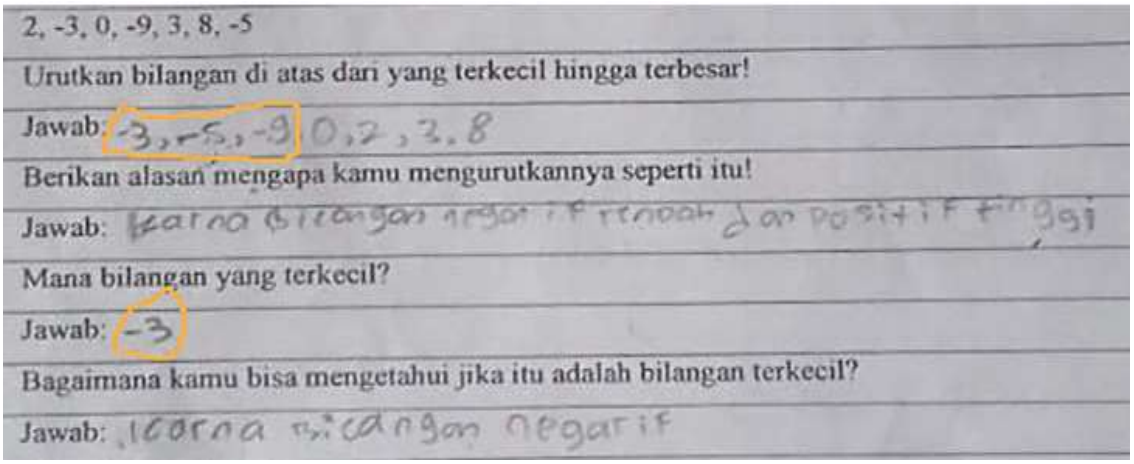

Figure 3. The response of the research subject on item 3

Translation:

Order the numbers above from smallest to largest!

Give your reason!

Which one is the smallest number?

How do you know that it is the smallest number?

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When the researchers asked why he sorted as he did, he answered that negative numbers are low and positive numbers are high. So, he writes negatives first and then writes positive numbers. Here is the interview excerpt.

T: "Why did you order the numbers so?"

S: "Yes the negative should be first because it's low, then positive because it's high."

T: "Why is the line like -3, -5, -9 and so on?"

S: "According to low numbers first, mom, 3, 5, and 9."

Based on the test results and interview excerpts above, the subject has not been able to sort between positive integers and negative integers. This is due to the subject's understanding that negative integers are low numbers so that in sorting negative integers, the subject rates equally when sorting two positive integers or regardless of the negative sign in front of the number.

When the subject was posed to contextual problems, the subject could not compare two negative numbers. This assumes the value of a negative number equals a positive number. Here is a snippet of the answer.

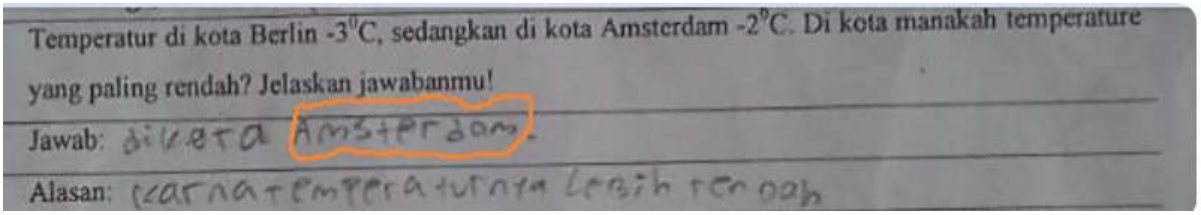

Figure 4. The response of the research subject on item 7

Translation:

The temperature in Berlin city is -3^0C, while in Amsterdam city is -2^0C. Which city has lower temperature between those two? Why?

The subject has not been able to compare two negative numbers, but he was able to solve the problem of rising temperatures. Based on this, S1was able to think about the symmetry of negative numbers and positive numbers. Here is a snippet of the answer.

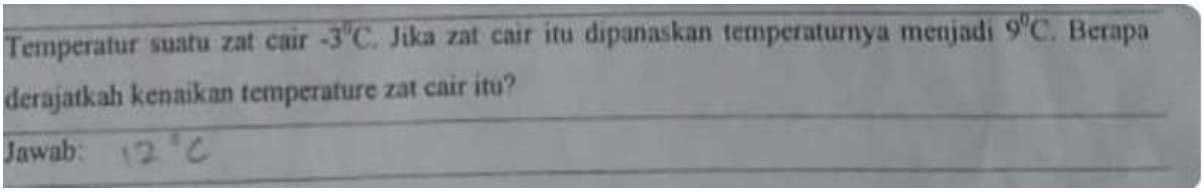

Figure 5. The response of the research subject on item10

Translation:

The temperature of the liquid is -3^0C. If the liquid is heated, the temperature is 9^0C. What is the degree of increase in the temperature of the liquid?

After the researchers asked why the subject could determine the result was 12, he responded with counts ranging from -2, -1, 0, 1, to 9 using his fingers. Here is the interview excerpt.

T: "How did you get this 12?"

S: "Like this, mom. -2, -1, 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 (counting with fingers), 12?"

Based on the results of the tests and interviews above, the subject was able to think about the symmetry of negative numbers and positive numbers to solve contextual problems of temperature increases. In this case, the subject does not view the negative integer as a low number, while a positive integer is a high number. But the subject solves contextual problems based on his knowledge of the symmetry values of integers.

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At the elementary school level, material of numbers is very important penting (Utami, Sa'dijah, Subanji, & Irawati, 2019). Before the researchers explain how formal mental models of elementary school students in understanding the concept of integers, the researchers will show how students understand the concept of integers. The results of the student's understanding of the concept of integers can be seen in the following table.

Table 2
The Students' Understanding about the Concept of Integers

Concept	The Subject's Understanding
Definition of negative integers and positive integers	A negative number is a low number with a "-" sign character (the subject calls it "min"), and a positive number is a high number. The subject also states that a positive number is greater than a negative number.
Comparing two numbers, which is a positive integer and a negative integer, and comparing two negative integers.	The subject was able to compare two negative numbers and positive numbers but could not yet compare two negative numbers. The subject ignores the "-" sign in front of the numbers 7 and 2. He considers the value of a negative number to be equal to a positive number.
Sorting integers	The subject has not been able to sort the negative numbers correctly. This looks at the value of a negative number as equal to a positive number. The subject holds a low negative number and a high positive number, so he writes the negative number first and then writes the positive number. The order of numbers from smallest to largest for negative integers is still incorrect, while the order of numbers from smallest to largest for positive integers is correct.
Comparing two negative numbers with contextual problems	The subject has not been able to compare two negative numbers on a contextual problem. This assumes the value of a negative number equals a positive number.
Contextual problems with rising temperatures	The subjects were able to solve the problem of temperature increase by counting from -2, -1, 0, 1 to 9 using their fingers.

Students do not always construct synthetic mental models of numbers after being exposed to the preceding facts. Students were unable to compare two negative numbers and to arrange negative integers and positive integers from smallest to greatest. This indicates that students are still in the transition mental model stage and have not progressed to the synthetic mental model stage. Students were able to answer the temperature rises from -3^0C to 9^0C by counting using their fingers with symmetry spanning from -2, -1, 0, 1, etc... Up to 9 and discovered 12 solutions. Students have progressed to the point of a formal mental model as a result of this.

The findings of this study indicated that some students lacked synthetic mental models of integers prior to showing formal mental models. In contrast to the findings (Bofferding, 2014), the study asserts that a significant proportion of students build synthetic mental models prior to demonstrating formal mental models.

The process of developing a formal mental model of the student's understanding of integers was by counting with his fingers from -2 to 9. While the student was unable to compare two negative scores, he was able to write negative integers from symmetry to zero and positive integers on line numbers. In line with (Case, 1996), 8-year-olds (3rd-grade primary school) are already in a two-dimensional stage of development necessary for them to compare two integers. Okamoto & Case (1996) stated that if a child can comprehend the values and sequences on a number line at the age of 8, he or she can develop an understanding of the broader number system by the age of ten.

CONCLUSION

The study's findings indicate that certain students lack the ability to build synthetic mental models of integers prior to showing formal mental models. For this reason, it is not always necessary for students to construct a formal mental model. Students who created formal mental models prior to establishing synthetic mental models were able to handle context-dependent integer tasks but were unable to compare or sort positive and negative integers.

The student's process of developing a formal mental model is evident when he answers the temperature increase problem by counting through his fingers and comparing the result to the position of zero symmetry between negative and positive integers. Although students have not yet been able to compare and sort integer numbers, they can now visualize rows of negative and positive integers using number lines.

The researchers propose that more study should be conducted to determine if students who build formal mental models prior to developing synthetic mental models would be able to gain a complete comprehension of topics in the future without encountering cognitive obstacles. Additionally, the researchers suggest that additional research should be conducted to determine why certain students generate synthetic mental models prior to demonstrating formal mental models. Why are there also students who do not develop synthetic mental models before?

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